Interferometry with Stabilization of Atmospheric Wavelength

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Abstract:
In this contribution we present an approach to incremental interferometric measurements of displacements over a limited range where the atmospheric wavelength of the coherent laser source is either directly stabilized to a mechanical reference or is corrected to fit to the reference. The idea comes from the possibility to use a highly stable material for a reference frame, material which can perform thermal expansion coefficients on the level $10^{-8}$/K within a large temperature range up to 50K. Over a range of several K which may be the practical range for displacement measurements the coefficient is even smaller. This may outperform the best techniques of correction for the variations of the refractive index of air. The mechanics is always a part of the measurement setup and represents one of the sources of uncertainty. A link of the refractive to the mechanical reference can practically eliminate another source of uncertainty.

Introduction
Interferometric measuring techniques with a highly coherent laser source has become a cornerstone for measurement of geometrical quantities in primary metrology, calibration of mechanical length standards and also in industrial applications where ultimate precision is needed. The overall concept is based on a highly stable laser source with stabilized optical frequency representing a standard of wavelength which is consequently seen as an elementary length counted by an interferometer. Further improvement of resolution of an interferometer below this length element has been achieved by a combination of optical techniques and advanced electronic digital signal processing of the interference signal. Stability of the optical frequency of laser sources which has been achieved recently is very precise. Traditional He-Ne lasers stabilized to the active Doppler-broadened line in Ne can operate with relative frequency stability on the level $10^{-8} – 10^{-9}$, He-Ne laser stabilized through subdoppler spectroscopy in iodine on the $10^{-11} – 10^{-12}$ level and the potential of iodine stabilized lasers based on frequency doubled Nd:YAG is very close to the $10^{-14}$ level [1]. The reproducibility of their absolute frequencies is another goal in metrology and is limited to $2.1 \times 10^{-11}$, resp. $9 \times 10^{-12}$ [2]. A conversion of a stable frequency into a precise wavelength relies on the value of
speed of light which is under vacuum conditions defined physical constant. In the laboratory environment the value of the refractive index of air has to be considered. The search and effort for a more precise interferometric measuring tool includes highly stable laser sources, reduction of noise, better optics, higher resolution through optical and electronic techniques, linearization, etc. [3, 4, 5, 6] Obviously, when measurement has to be performed on air – in the routine laboratory measurements – the refractive index of air represents a major source of uncertainty.

In the laboratory environment where the calibrations and comparisons of interferometers are performed interferometric refractometer is necessary. The simplest configuration is a differential interferometer measuring with high resolution the difference between an air and vacuum path within defined distance of an evacuated cell [7, 8]. Other approaches of refractometer design are trying to be compact and at the same time to find a more precise way of measurement where the value of refractive index is available on-line or at least more often than once the cell is evacuated and filled again. The systems include movable triangular cells, flexible cells that could be elongated, and some others arrangements [9, 10].

All measurements of the refractive index of air performed by refractometers or by evaluation of the Edlen formula suffer one principal limit which is the fluctuations of air along and around the laser beam axis. Furthermore, there are always thermal gradients present in the air – mainly in the vertical direction. The sensors, primarily thermal, can be placed close to the beam. However, not directly into the laser beam. Moreover, only selected points can be measured. Laser beam of the refractometer can be again placed only close to the measuring path. While the evaluation of the refractive index of air through direct refractometry under laboratory conditions can be done with the uncertainty close to the $10^{-9}$ [11, 12, 13] the limiting factor seems to be the stability of the atmosphere around the beam path. The practical limit for determining the refractive index of air is determined by the thermal gradients and air fluctuations that can be avoided depending on the application.

In case of all commercial interferometric systems the compensation of index of refraction of air is done by measuring of the fundamental atmospheric parameters – temperature, pressure and humidity of air, accompanied in some cases by the measurements of concentration of carbon dioxide. The value of refractive index is extracted by evaluation of empirical Edlen formula [14, 15, 16, 17]. Thus, the limits of this indirect determination of the refractive index are primarily given by the configuration of the measuring setup, by the air flow and stability of atmospheric conditions close to the laser beam rather than by the precision of sensors measuring temperature, etc. or the formula itself.
The effort to combine the distance measuring interferometer and the refractometer into one instrument which could evaluate the influence of the refractive index of air during the measurement or directly compensate for it. There were arrangements presented where a complex set of two separate interferometers evaluate the refractive index of air and measure the distance [18]. This system can compensate for the refractive index but is unable to overcome the problem of the determination of the refractive index in the laser beam axis. A method linking the wavelength of the laser source to the mechanical length of some frame or board was proposed by [19]. Authors suggest using a set of two identical interferometers where one is fixed in the length and serves as a reference for the laser wavelength. Other approaches represent a completely different methods for determination of the refractive index of air, for example through the speed of sound at ultrasonic frequency range [20, 21]. Also, the control of the refractive index which is kept constant was suggested [22].

**Over-determined interferometer**

To get the information about the actual value of the refractive index of air directly in a tracking refractometric regime together with interferometric measurement of displacement means to assemble at least two interferometric systems. They may be placed next to each other or they may share the beam path. Merging these two instruments results in an arrangement where the displacement is measured as an overdetermined quantity, for example from two directions in a countermeasuring setup (Figure 1).

![Fig. 1: Principal schematics of the interferometric system with two countermeasuring Michelson interferometers. Ia, Ib: interferometers, M: movable flat mirror, F: fiber-optic light delivery from a single laser source, B: baseplate, La, Lb: displacements measured by the interferometers.](image)

In case of measurement of displacement there has to be always stated what is measured against what. All the traditional sources of error (refractive index, vibrations, laser noise, li-
nearity, misalignment, etc.) can be marginal when there is a mechanical instability of the reference point. The approach we present here combines the mechanical referencing of the interferometer itself with referencing of the laser wavelength. The mechanical referencing simply cannot be avoided so we at least link another source of variations (refractive index) to another (mechanical). Displacement measuring interferometer and refractometer are not clearly divided which beam measures the refractive index of air and which one the displacement. From one point of view it may be a question which length from the two measured in Figure 1 should be the correct one. From another one it is not possible to say, both are relevant or the stability of the frame may is the key parameter. The solution presented here suggests measurement in one axis that is the measurement axis both for evaluation of the refractive index of air and the measured length. There may be differences in refractive indices of the left and right measured paths (Figure 1) but expressing the resulting displacement as a mean value of these two reflects the concept that neither of these two can be seen as the “correct” one. In this case the refractometer and interferometer not only become one instrument but the beam paths of these two are identical. This can be seen as the major advantage of this concept reducing or even eliminating the sources of error arising from differences in measured refractive index and refractive index in the measuring path of the interferometer. When the wavelength (or mean wavelength) is fixed over the whole measuring range and the wavelength control loop is fast the system can adequately react even to fast changes of refractive index caused by turbulations or air flow. The question of dead length is not so relevant here while the stabilization of wavelength works in the whole range set by the position of the two interferometers. Obviously to set the range rather as short as needed is better and reduces the influence of variations along the beam path and between the left and right measured displacements.

Stabilization of wavelength on air over the measuring range means real trekking of the refractive index fluctuations. There is a clear limit of laser tuning range which limits the range of the refractive index variations. Considering temperature the greatest source of its change the drift over 1K needs laser frequency tuning approx. over 1 GHz. This is a maximum of single frequency He-Ne laser with homodyne detection scheme. Choice of other laser sources would be needed for practical use. Suitable frequency-doubled Nd:YAG or DFB laser diode might be the option. Thermal tuning may on the other hand limit the time response and ability to reflect fast changes of the refractive index.

Much more attractive can be the chance to keep the laser frequency stabilized some traditional way and to do the compensation of the refractive index drift through on-line calculation. In this case the absolute length of the measuring range, resp. the air measuring beam path in
both interferometers must be known. Drift of the sum value (La + Lb) when used to derive the control signal for wavelength locking is enough, no matter how large it is in the absolute value. To recalibrate the measured displacement from either left or the right interferometer keeping the laser frequency constant we need to know the relative change of the wavelength.

**Setup configuration**

The principal configuration in Figure 1 with a flat mirror interferometer needs both interferometers fixed to a reference frame – baseplate made of material with low thermal expansion. The same way the moving mirror thickness counts as well contributing to the overall length measured over the given range. A flat, solely reflecting mirror made of the low thermal expansion material as well looks like a pure solution. Sensitivity to the straightness of its motion is a significant disadvantage introducing cosine errors. Non-equal lengths La and Lb produce different errors measured by both interferometers.

The configuration with the stabilization of the laser wavelength can be seen also as a de facto standing-wave interferometer. To put it together directly this way might be an attractive option either simply with two counterpropagating beams or in a cavity-like setup. This approach needs a component able to track the interference maxima and minima along the beam axis. In [23, 24, 25] a transparent photodetector has been reported even in a design with two active domains separated by a distinct spacing suitable for generation of quadrature signals usual in displacement interferometry. Suitable balance between the losses caused by the detector to the beam passing through and its sensitivity has to be found when it should be placed into a passive resonant cavity (Figure 3).

**Fig. 2: Configuration with a passive Fabry-Perot cavity.** M: cavity mirror, PD: photodetector, TPD: transparent photodetector, F: fiber-optic light delivery, B: baseplate, La, L: displacement and overall length.
The transparent photodetector when deposited in a form of a thin film on a glass substrate contribute to the measuring length in a negligible way, the glass substrate on the other hand has to follow the need of small thermal expansion coefficient. We designed a detector as photoresisitive silicon coating with conductive electrodes on both sides. This reduces the losses while only the silicon layer is in the beam path. Photo of an experimental detector is in Figure 3.

![Photo of an experimental detector](image)

Fig. 3: Photoresistive detector, Si coating deposited on a glass substrate with side electrodes.

The link between the wavelength and mechanical reference here is in principle simple, the laser optical frequency has to be locked to the resonance of the passive cavity either through tracking the transmission maximum or reflection minimum. In this case the approach of laser optical frequency control over a specific range is the only option. The tuning range has to cover the entire range of possible variations of the refractive index of air.

**Experimental testing**

To prove the principle we assembled first the most simple configuration from Figure 1 on a baseplate made of “0” grade Zerodur. We applied the tracking of the refractive index of air through laser optical frequency tuning in a feedback control loop. The experiment was designed to compare the laser detuning with values of refractive index measured indirectly by evaluation through traditional Edlen formula. A set of sensors monitoring temperature, humidity, pressure and content of the carbon dioxide was inserted into the thermal controlled box.

The experiment has been performed in a static regime with the movable mirror in a fixed position in the center position between the two interferometric units [26]. Thermal control of the environment inside allowed continuous rise and fall of the refractive index. The thermal shift causing associated shift of the refractive index was ca. 1K, the limiting factor being the mode-hop free tuning range of the He-Ne laser source. The overall relative change in the optical path expressed through laser tuning over 900 MHz was $9.2 \times 10^{-7}$. The value to com-
pare with was the change of the refractive index $8.7 \times 10^{-7}$ evaluated through indirect measurements of the parameters of atmosphere and calculation using Edlen formula.

In any interferometric measurements the mechanics seems to be the greatest or at least a significant source of error. Either in a form of acoustically generated noise and vibrations or through thermal effects influencing the dimensions of the components. In our case we designed the interferometric units as fully thermally compensated having identical reference and measuring paths in the glass and we cared for referencing them to the baseplate. Still the units consisted of SF 14 beamsplitters, quartz glass retardation plates, all housed in Titanium alloy with thermal expansion fitted to the beamsplitters. The thermal drift generated in the box was measured in the air so the thermal drift of the components followed with some delay. Thus the agreement at the level of $5 \times 10^{-8}$ can be considered a proof of principle and close to the limit achievable under stable laboratory conditions considering the microturbulations of the air inside.

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